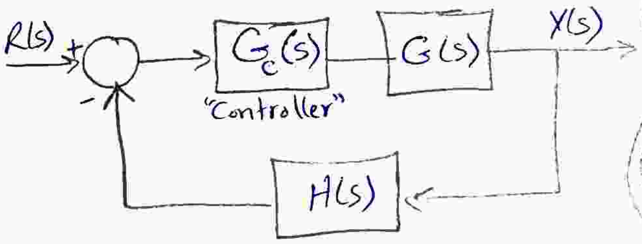
Design of Controllers

→ Controllers used to improve system performance

- → sys. dynamic (overshoot, t_r, t_s, ...)
- → steady-state error

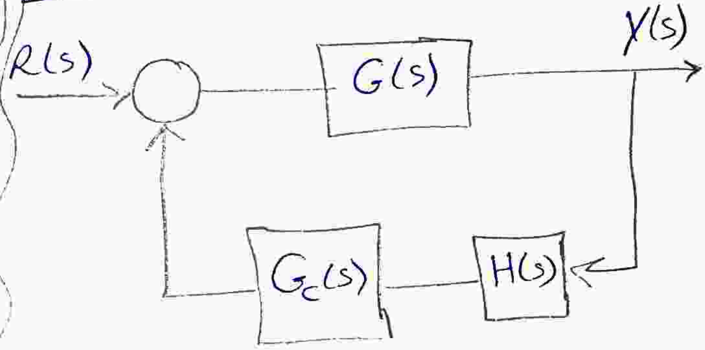
⇒ design of controllers

1) Compensated system



↳ the most used one

2



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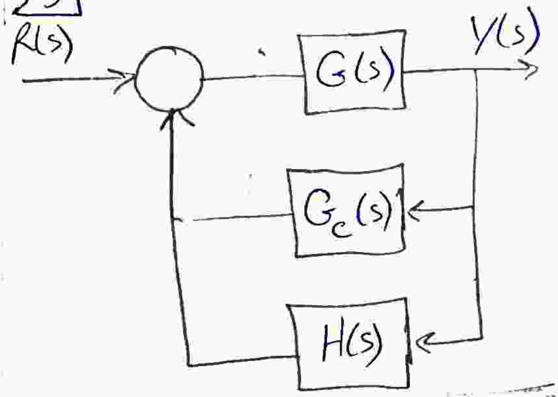

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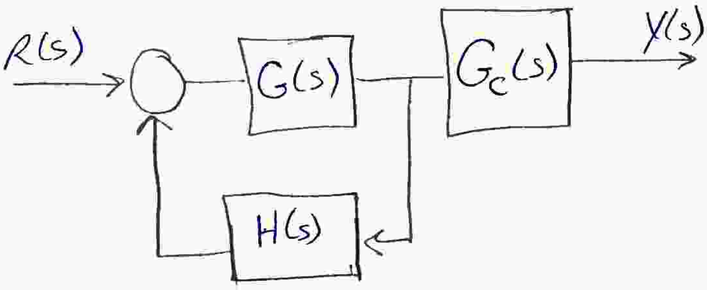

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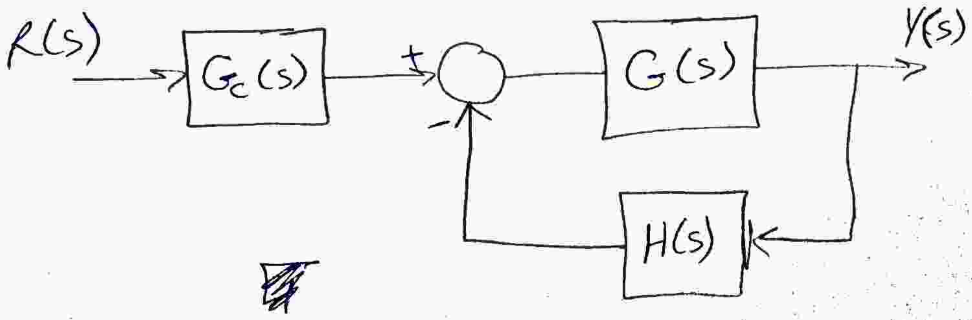


[1] classical Controllers (traditional)

* PI Controller ⇒ improve steady state error

* PD Controller ⇒ improve system dynamics
  - → speed up system response & reduce overshoot.
  - → reduce the part of transient.

* PID ⇒ balance between PD & PI.

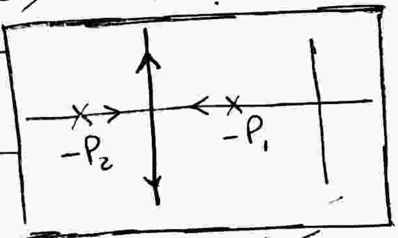
* Phase-lag ⇒ the same as PI.

* Phase-lead ⇒ " " " PD.

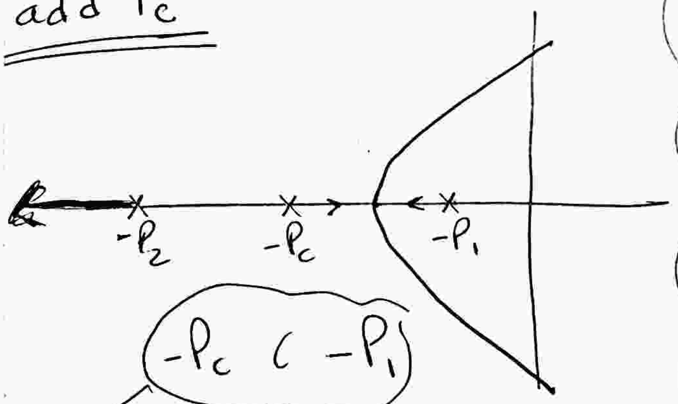
* Phase lead-lag ⇒ " " " PID.

← باقي التوصيفات للمعرفة فقط ومتواجدة في Lec 7

Phase-lead Controller

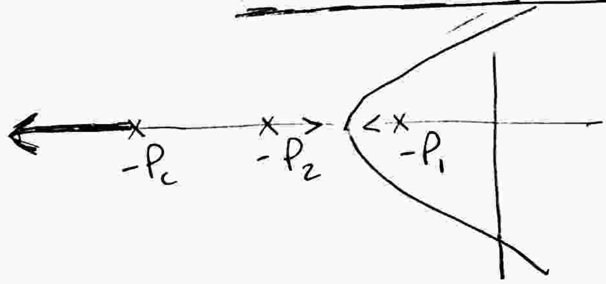


add P_c



$-P_c < -P_1$

→ 2-dominent Poles.



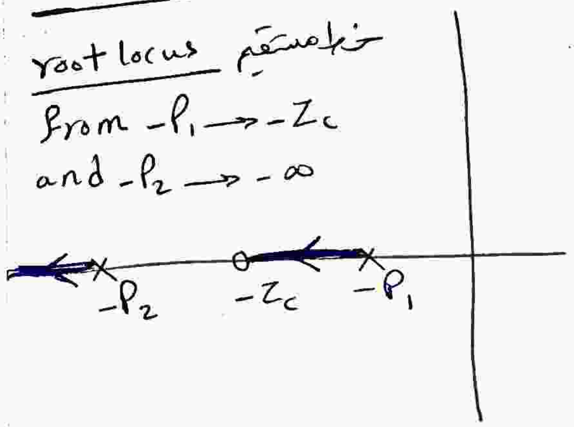
$-P_1, -P_2$ → two dominent poles

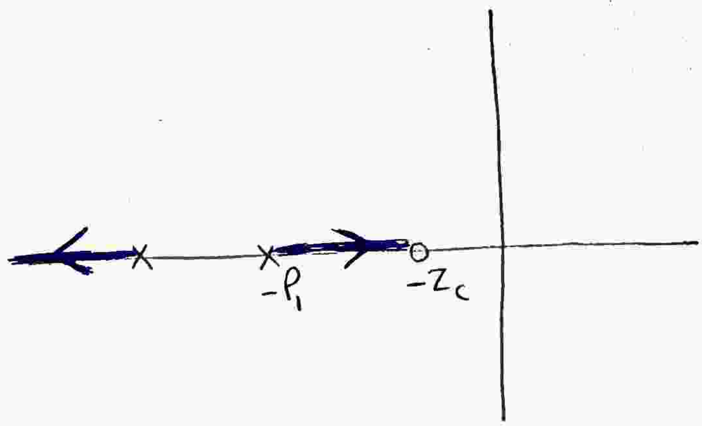
add zero

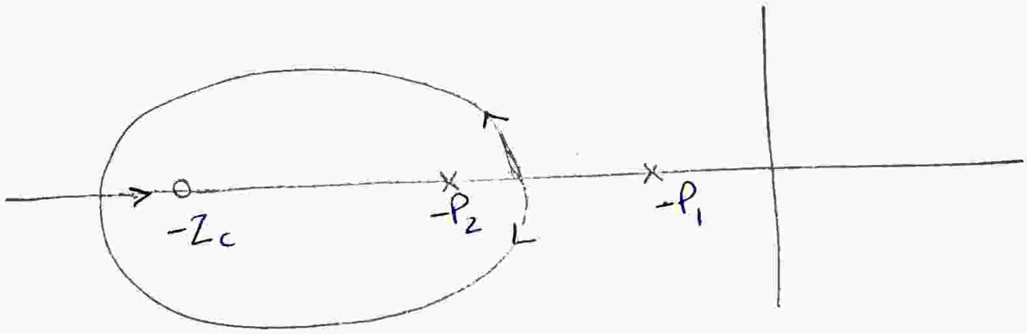
root locus خط مستقیم

from $-P_1 \rightarrow -Z_c$

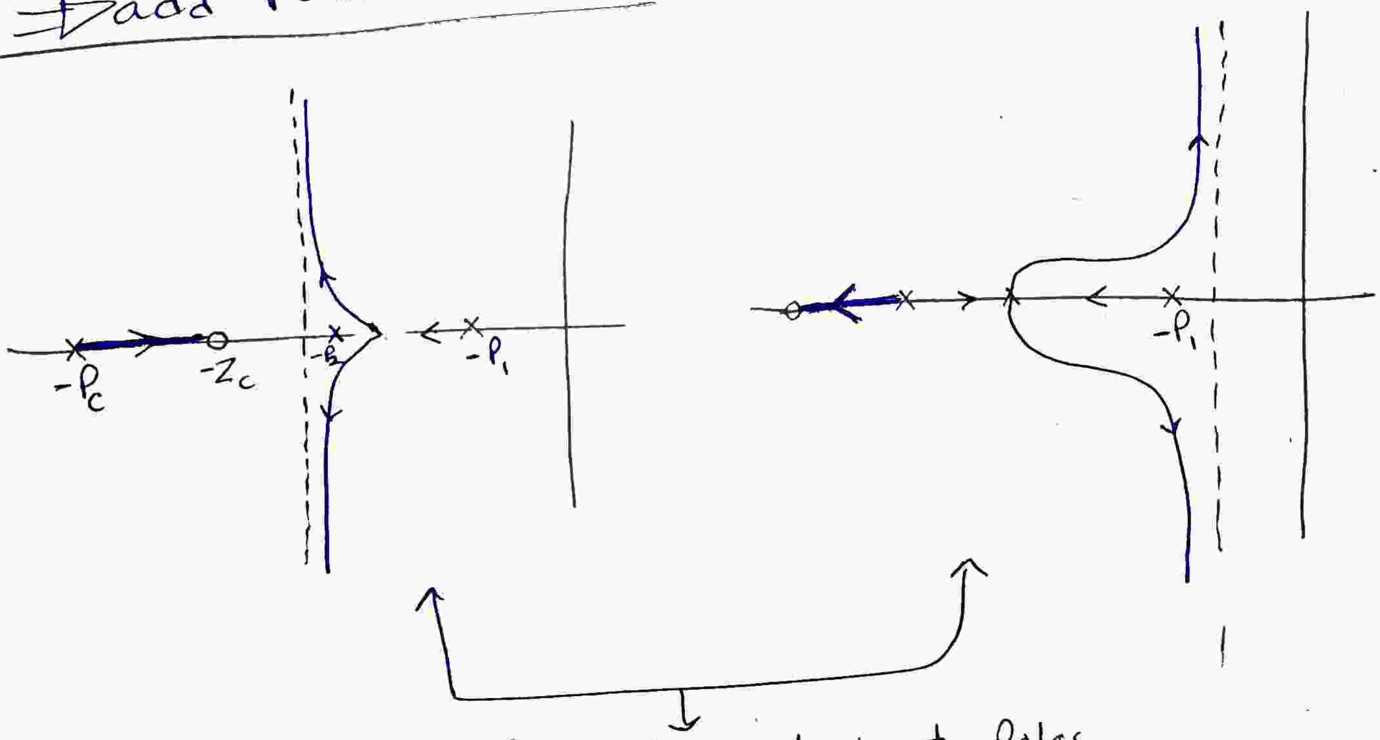
and $-P_2 \rightarrow -\infty$





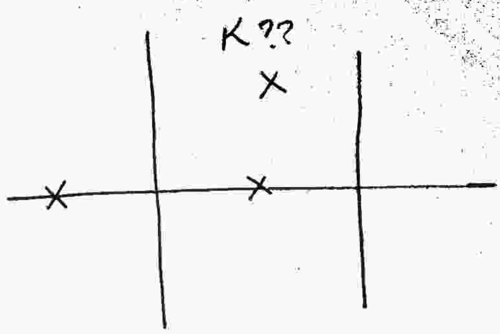


⇒ add Pole and zero



$-P_1, -P_2 \rightarrow$ two dominent Poles

← لو طلب K عند النقطة دى
فهى غير منتمية لـ (root locus)
لما ضفت pole ، zero جعلت
من الممكن الـ (root locus) يمر بيها.



$$G_c(s) = \frac{s+Z_c}{s+P_c}$$

- $|Z_c| < |P_c| \Rightarrow$ Phase-lead.
- $|P_c| < |Z_c| \Rightarrow$ Phase-lag

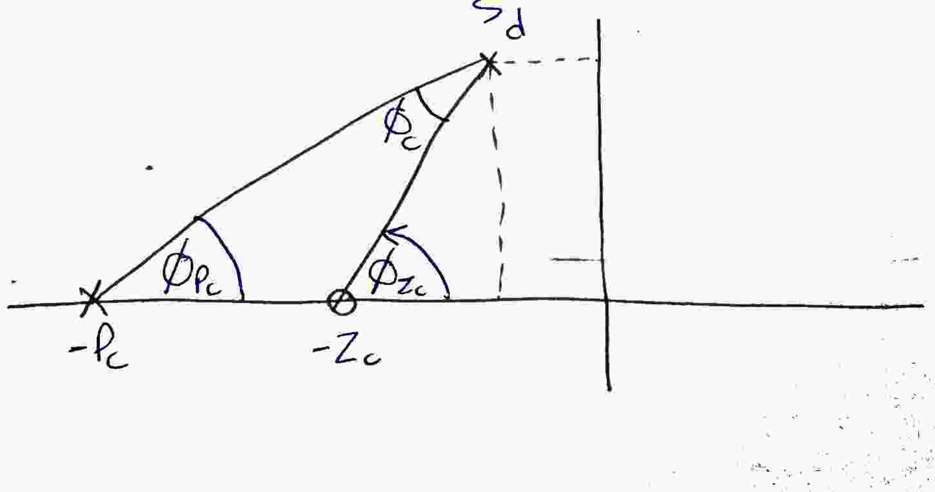
→ there is example for electrical system in Lec 7 page 12, 13, 14.

⇒ The design steps to find P_c, Z_c

$\boxed{1}$ using given required specs (ζ, ω_n, t_r, M_p, ---) to obtain location of desired poles

$$S_{d1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$\boxed{2}$ Apply angle condition to obtain compensator angle (ϕ_c)

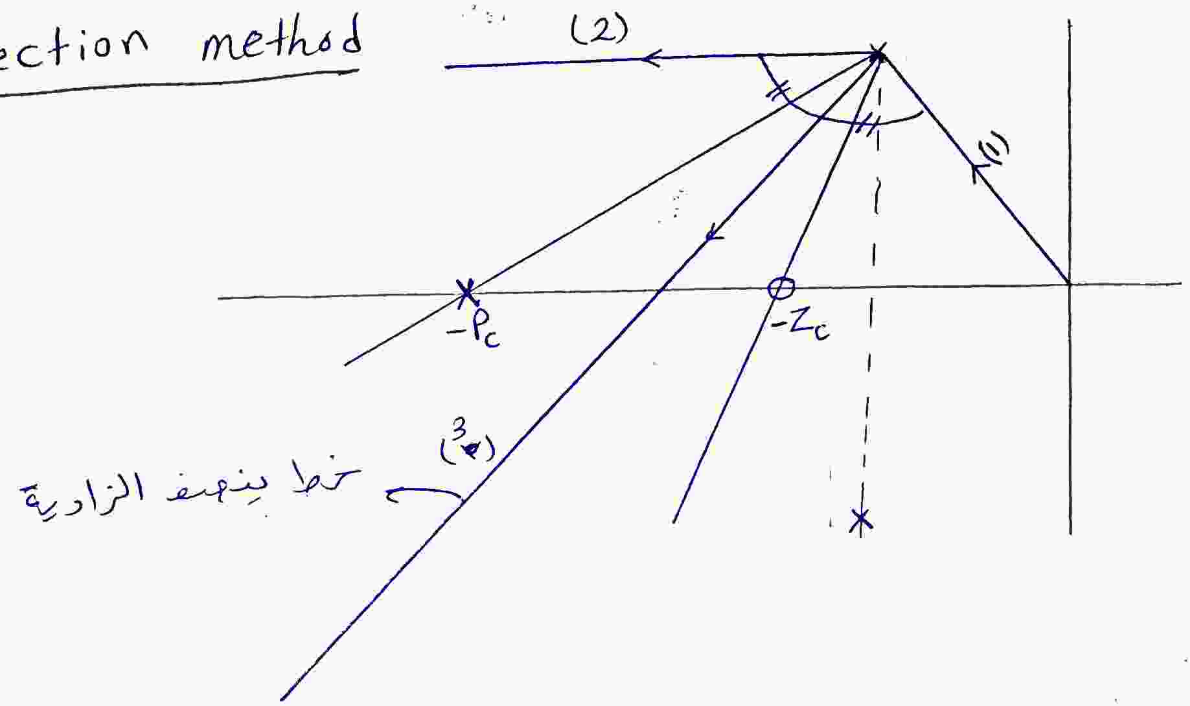


$$\angle GH + \phi_c = -180$$

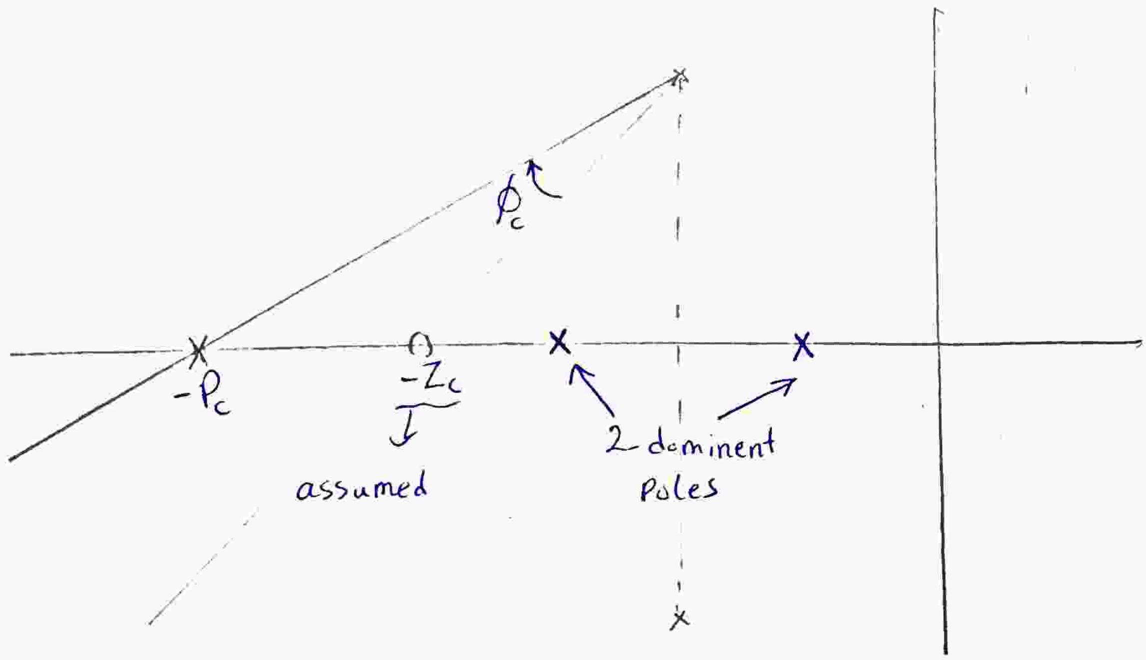
$$\phi_c = \phi_{Z_c} - \phi_{P_c}$$

$\boxed{3}$ determine location of Z_c, P_c by Know of ϕ_c

a) Bisection method



b)

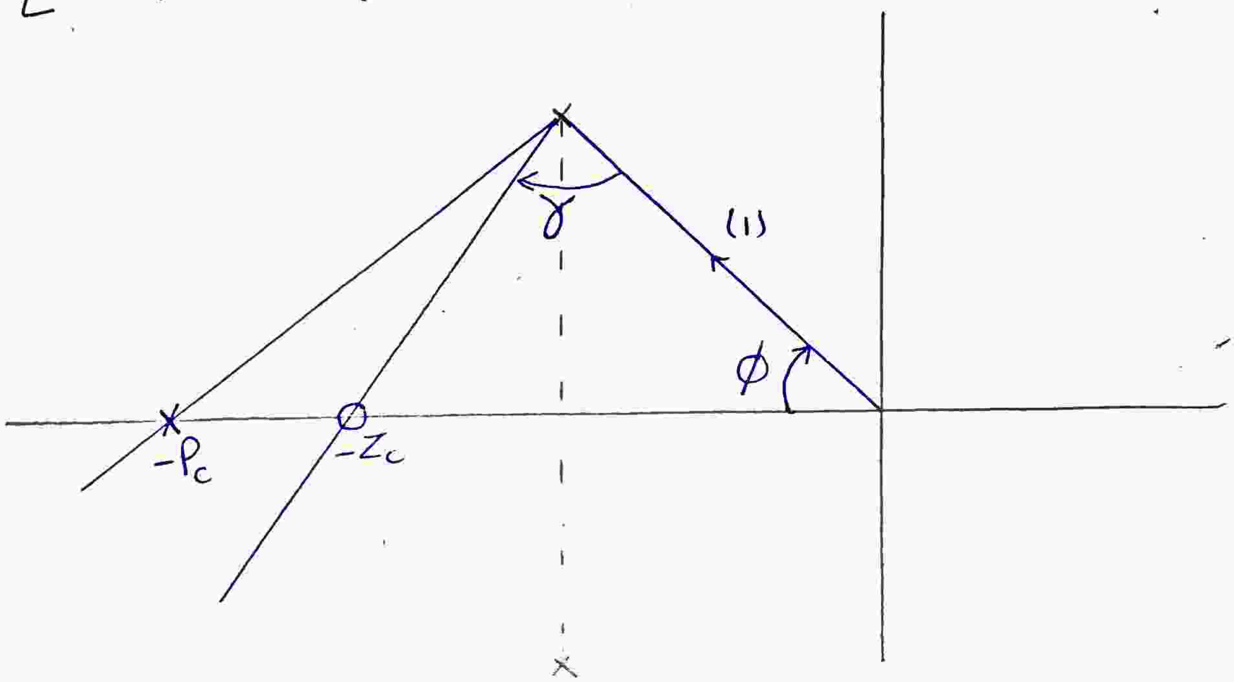


ـــ يفترض مكان ال (Zero) على شمال ال (2nd Pole)

من ال (2-dominent Poles)

$\boxed{C}$ Max. attenuated ratio

$$\gamma = \frac{1}{2}\left[\pi - \phi - \phi_c\right] \quad ; \quad \phi = \cos^{-1} \zeta$$



$\boxed{4}$ "in design steps"

Apply magnitude Condition to determine the value of gain K to meet desired specs:-

$$\left\| K\, G_c(s)\ GH(s) \right\|_{s=s_d} = 1$$

<u>or</u>
$$K = \frac{\prod \text{Poles}}{\prod \text{Zeros}}$$

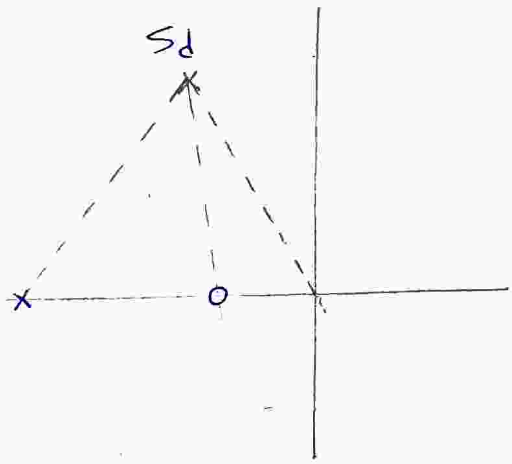
For checking steady-state error (e_{ss})

$* \; r(t) = 1 \Rightarrow K_p = \lim_{s \to 0} GH(s) \Rightarrow e_{ss} = \frac{1}{1+K_p}$

$* \; r(t) = t \Rightarrow K_v = \lim_{s \to 0} s\,GH(s) \Rightarrow e_{ss} = \frac{1}{K_v}$

$* \; r(t) = \frac{t}{2} \Rightarrow K_a = \lim_{s \to 0} s^2\,GH(s) \Rightarrow e_{ss} = \frac{1}{K_a}$

$\Rightarrow$ Lead Compensator



at S_d: $\sum \phi_z - \sum \phi_p \neq \pm 180$

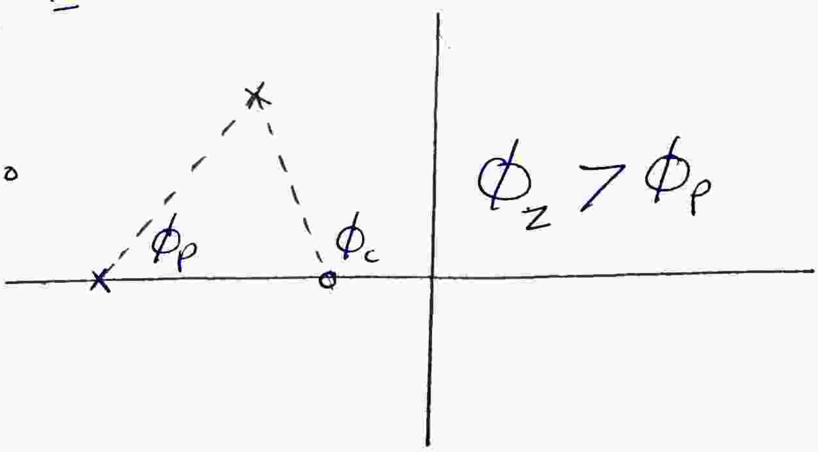
desired

at S_d: $\sum \phi_z - \sum \phi_p = \pm 180$

$\hookrightarrow$ By adding pole & zero to the system

$\sum \phi_{z_c} - \sum \phi_{p_c} = \pm 180$

$\angle GH(s) + \phi_c = -180$



$\phi_z > \phi_p$

* lag Compensator design

1) Calculate

$$\beta = \frac{K_c}{\underbrace{K_{un}}_{\text{unctrolled}}} * \underbrace{1.1}_{\text{safety factor}}$$

K_c : desired value of dc gain

K_{un} : system value of dc gain

2) Assume Zero location (Z_c) in range of 10% from the 2nd diminent pole of the sys

$$Z_c = \frac{10}{100} * P_2 \qquad P_2 \Rightarrow \text{2nd diminent pole.}$$

$$P_c = \frac{Z_c}{\beta} \qquad \underbrace{K_{dc}}_{\text{of controller}} = \frac{Z_c}{Z_c/\beta} = \beta$$

state space

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

$D=0 \Leftarrow$ درجة البسط في ال (T.F) أقل من درجة المقام.

$A_{n\times n} \Rightarrow$ system matrix , $B_{n\times 1} \Rightarrow$ i/p matrix

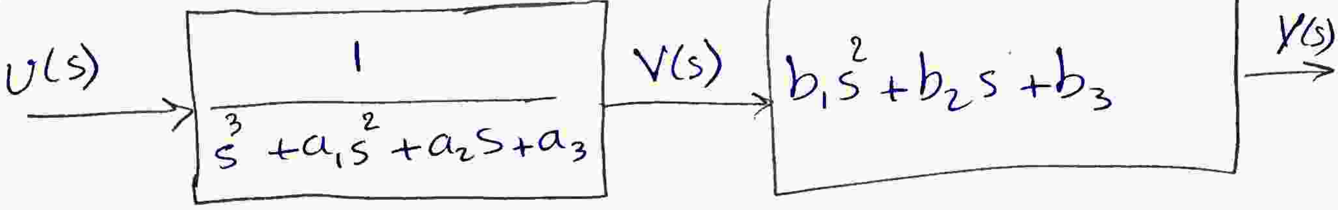
$C_{1\times n} \Rightarrow$ o/p matrix $\quad x(t)_{n\times 1} \Rightarrow$ state vector

Canonical forms for state space

[1] Controllable form

For 3rd order system $\quad T.F = \dfrac{Y(s)}{U(s)} = \dfrac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$

a)



← كملها هتوصل للمصفوفات.

b) طريقة مختصرة

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

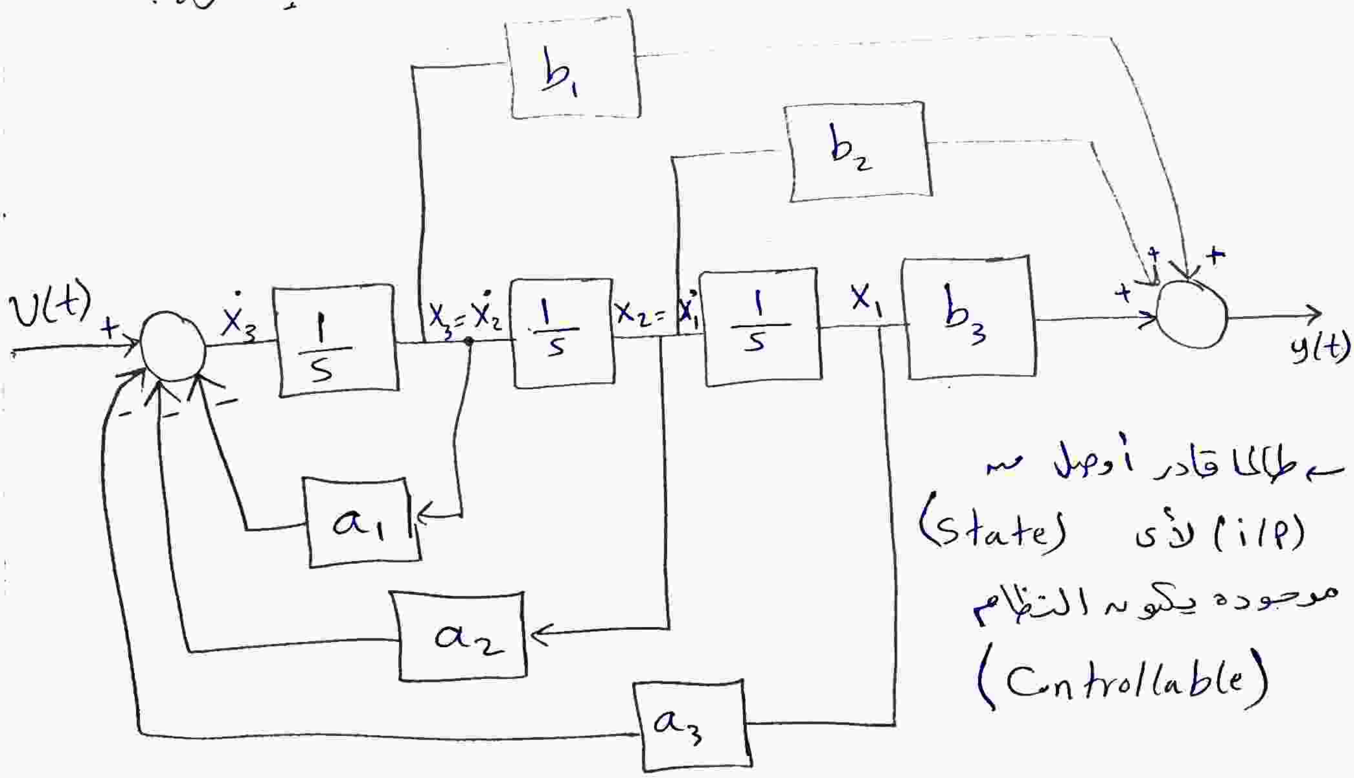
First check that Coeff. of $s^3 = 1$

↳ أكبر أس

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}}_{\text{معاملات المقام بإشارة معكوسة}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

1 ⟶ معامل s^3

$$y(t) = \underbrace{(b_3 \quad b_2 \quad b_1)}_{\text{معاملات البسط معكوسة لكن بنفس الإشارة}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



← طالما قادر أوصل من (i/p) لأي (state) موجودة يكون النظام (Controllable)

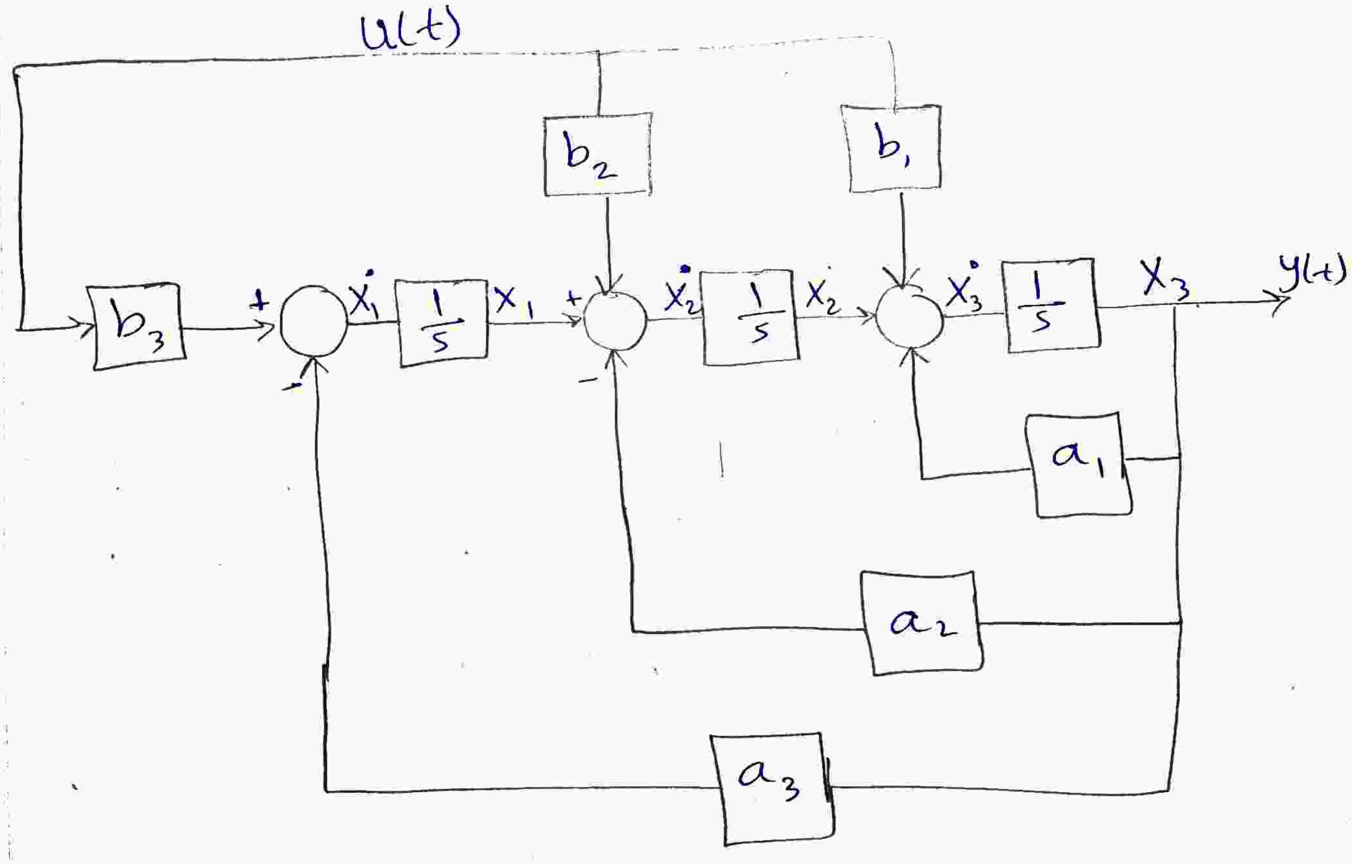
2 observable form

ex

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{pmatrix} x(t) + \begin{pmatrix} b_3 \\ b_2 \\ b_1 \end{pmatrix} u(t)$$

$$y(t) = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x(t)$$



3 Diagonal form

if

$$T.F = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{(s+P_1)(s+P_2)(s+P_3)} = \frac{A_1}{s+P_1} + \frac{A_2}{s+P_2} + \frac{A_3}{s+P_3}$$

$$Y(s) = \underbrace{\frac{U(s)}{(s+P_1)}}_{X_1(s)} \cdot A_1 + \underbrace{\frac{U(s)}{(s+P_2)}}_{X_2(s)} \cdot A_2 + \underbrace{\frac{U(s)}{(s+P_3)}}_{X_3(s)} \cdot A_3$$

$$X_1(s) = \frac{U(s)}{s+P_1} \Rightarrow U(s) = (s+P_1)\,X(s)$$

$$u(t) = \dot{x}_1(t) + P_1 x(t)$$

$$\dot{x}_1 = -P_1 x_1 + u(t) \quad ; \quad \dot{x}_2 = -P_2 x_2 + u(t)$$

$$\dot{x}_3 = -P_3 x_3 + u(t)$$

$$\dot{x}(t) = \begin{bmatrix} -P_1 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -P_3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t) \quad \Rightarrow (1)$$

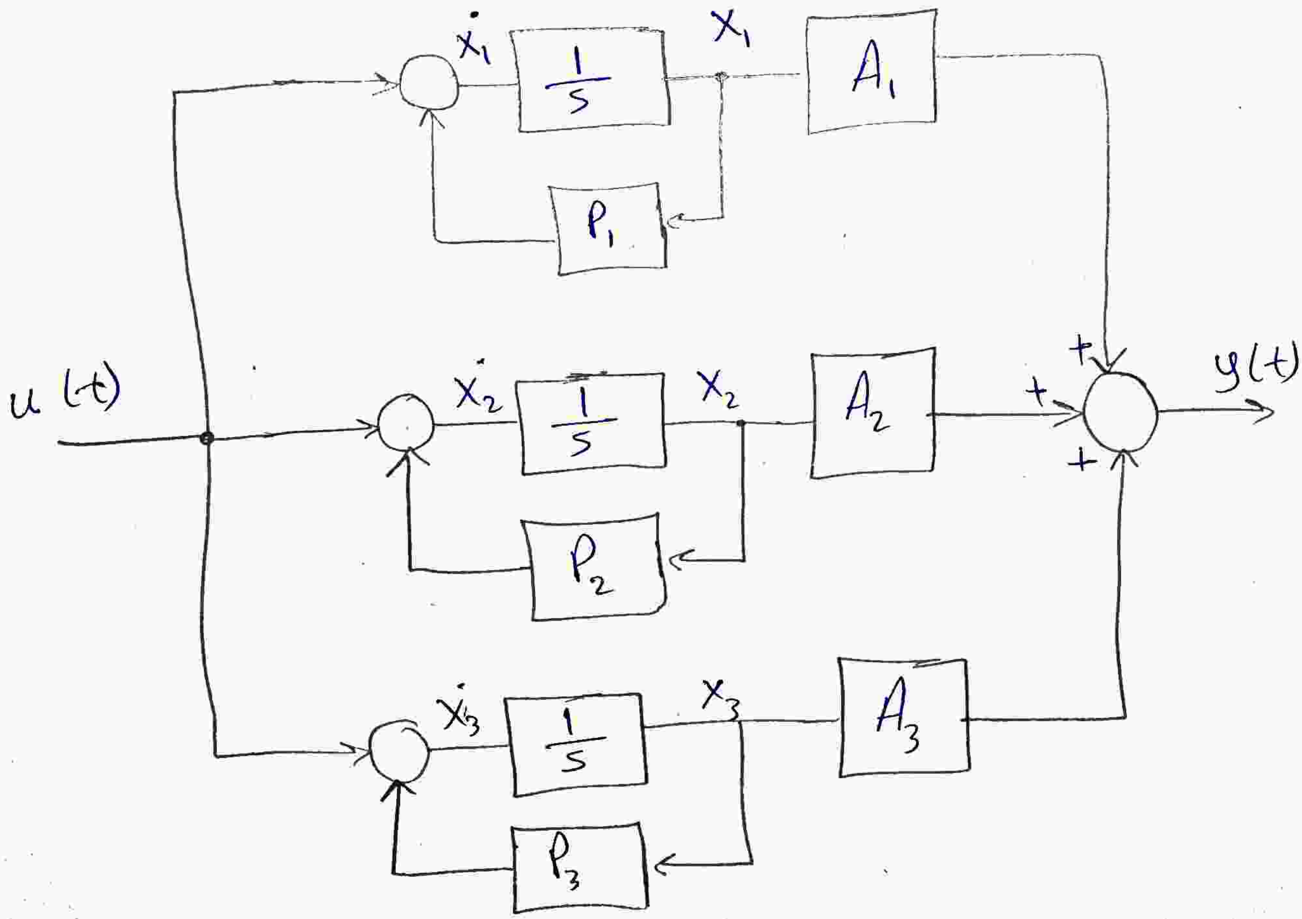
$$y(s) = A_1 X_1(s) + A_2 X_2(s) + A_3 X_3(s)$$

$$y(t) = A_1 x_1(t) + A_2 x_2(t) + A_3 x_3(t)$$

$$y(t) = \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} x(t) \longrightarrow (1)$$

For repeated Poles

$$\dot{x}(t) = \begin{bmatrix} -P_1 & 1 & 0 \\ 0 & -P_1 & 0 \\ 0 & 0 & -P_2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$



State-space Analysis

Given

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

← تفاصيل النقطات الثلاثة القادمة في المحاضرات.

1 T.F

$$T.F = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B$$

2 Ch. equation

$$|sI-A| = 0$$

← تنتج معادلة جذورها هي الـ (Poles) بتاعة الـ (system).

3 System response to i/p u(t)

if $x(0) \neq 0 \Rightarrow \dot{x}(t) = Ax(t) + Bu(t)$ ↓ $\mathcal{L}.T$

$$sX(s) - AX(s) = BU(s) + x(0)$$

$$(sI-A)X(s) = x(0) + BU(s)$$

$$X(s) = (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s)$$

Let $\Rightarrow \phi(s) = (sI-A)^{-1} \Rightarrow$ transition matrix

$$\boxed{X(s) = \phi(s)\,x(0) + \phi(s)\,B\,U(s)}$$

$y(t) = C\,X(t) \Rightarrow Y(s) = C\,X(s)$

$Y(s) = C\left[\phi(s)\,x(0) + \phi(s)\,B\,U(s)\right]$

$Y(s) \xrightarrow{\mathcal{L}^{-1}T} y(t)$

$\boxed{4}$ Controllability

→ if system states change by changing system i/p.

→ controllability matrix (M_c)

على حسب درجة ال (system)

$M_c = (B \quad AB \quad A^2B \ldots \quad A^{n-1}B)$

⇒ if $|M_c| \neq 0$ → system is Controllable

2nd order ⇒ $M_c = (B \quad AB)$

3rd order ⇒ $M_c = (B \quad AB \quad A^2B)$

$\boxed{5}$ observability

← لو عندك (states) ومش عارف تعملها (estimation)

نستخدم ال (observer)

← لو عارف الخرج وعايز تعرف ما بداخل النظام

في الحالة دي النظام يكون (observable)

** In some cases, the states couldn't be measured for the following reasons:-

1- the location for physical states.

2- The measuring instruments are not valid.

$\rightsquigarrow$ if internal states can be calculated from observation of o/p response $\Rightarrow$ system is observable

$\Rightarrow$ observable matrix M_o

$$M_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} : |M_o| \neq 0 \quad \text{observable}$$

Lec 10, 11 باقي